VCE Specialist Mathematics Formula Sheets

Original worked revision examples with notes on units and assumptions.

Use the board PDF below for the official sheet. Coverage of examples varies by subject.

Official reference material

Official VCAA Examination 1 formula sheet

Official VCAA Examination 2 formula sheet

What to work out beyond the sheet

When a question asks for a principal argument, first find the correct quadrant and then choose the equivalent angle in the required interval.

Formulas and worked examples

Mensuration · circle segment

Formula or reference: $A = r^2(\theta - \sin\theta)/2$

Units and conditions: θ in radians and A in square units.

Worked example: For $r = 2$ cm and $\theta = \pi/2$, $A = (\pi - 2)$ cm², the quarter-circle area less the triangle area.

Mensuration · cylinder, cone, pyramid and sphere

Formula or reference: Cylinder: $V = \pi r^2 h$. Cone: $V = \pi r^2 h/3$. Pyramid: $V = Ah/3$. Sphere: $V = 4\pi r^3/3$.

Units and conditions: Use perpendicular height h, base area A and cubic units.

Worked example: At $r = 2$ cm and $h = 3$ cm, the cylinder has volume 12π cm³ and the cone 4π cm³. A pyramid with $A = 12$ cm² and $h = 3$ cm has volume 12 cm³. A sphere of radius 3 cm has volume 36π cm³.

Trigonometry · triangle formulas

Formula or reference: $A = bc \sin A_0/2$, where A_0 is the included angle. $a/\sin A_0 = b/\sin B = c/\sin C$. $c^2 = a^2 + b^2 - 2ab\cos C$.

Units and conditions: Match lengths to their opposite angles. Use square units for area.

Worked example: A right triangle with legs 3 cm and 4 cm has area 6 cm² and hypotenuse 5 cm by the cosine rule. Its angle opposite the 3 cm leg satisfies $\sin A_0 = 3/5$.

Complex numbers · modulus

Formula or reference: $|x + iy| = \sqrt{(x^2 + y^2)}$

Units and conditions: The modulus is non-negative.

Worked example: For $z = -3 + 4i$, $|z| = \sqrt{(9 + 16)} = 5$.

Complex numbers · polar form and principal argument

Formula or reference: $z = x + iy = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$. For $z \neq 0$, $-\pi < \mathrm{Arg}(z) \leq \pi$.

Units and conditions: $r = |z|$ and θ is in radians. $\mathrm{Arg}(z)$ denotes the principal value.

Worked example: For $z = -1 + i$, $r = \sqrt{2}$ and $\mathrm{Arg}(z) = 3\pi/4$, giving $\sqrt{2}\,\mathrm{cis}(3\pi/4)$.

Complex numbers · products and quotients

Formula or reference: $z_1z_2 = r_1r_2\,\mathrm{cis}(\theta_1+\theta_2)$. $z_1/z_2 = (r_1/r_2)\,\mathrm{cis}(\theta_1-\theta_2)$ for $z_2 \neq 0$.

Units and conditions: Use the same angle units for both factors.

Worked example: For $z_1 = 6\,\mathrm{cis}(\pi/3)$ and $z_2 = 2\,\mathrm{cis}(\pi/6)$, the product is $12i$ and the quotient is $3\,\mathrm{cis}(\pi/6)$.

Complex numbers · powers

Formula or reference: If $z = r\,\mathrm{cis}\,\theta$, then $z^n = r^n\,\mathrm{cis}(n\theta)$ for integer n, with $z \neq 0$ for negative n.

Units and conditions: Use the same angle convention throughout.

Worked example: $(2\,\mathrm{cis}(\pi/6))^3 = 8\,\mathrm{cis}(\pi/2) = 8i$.

Statistics · linear transformations

Formula or reference: $E(aX+b) = aE(X)+b$. $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$.

Units and conditions: Shifting changes the mean but leaves variance unchanged. Variance uses squared units.

Worked example: If $E(X) = 4$ and $\mathrm{Var}(X) = 9$, then $2X+3$ has mean 11 and variance 36.

Statistics · weighted and independent sums

Formula or reference: $E(\Sigma a_iX_i) = \Sigma a_iE(X_i)$. For independent variables, $\mathrm{Var}(\Sigma a_iX_i) = \Sigma a_i^2\mathrm{Var}(X_i)$.

Units and conditions: Independence is needed for this variance rule. The expectation rule is linear regardless.

Worked example: For independent X and Y with means 3 and 5 and variances 2 and 4, $2X-Y$ has mean 1 and variance 12.

Statistics · identically distributed observations

Formula or reference: For independent identically distributed X_i with mean μ and variance σ^2, $E(\Sigma X_i) = n\mu$ and $\mathrm{Var}(\Sigma X_i) = n\sigma^2$. $E(\bar{X}) = \mu$ and $\mathrm{Var}(\bar{X}) = \sigma^2/n$.

Units and conditions: Use n independent observations with common mean and finite variance.

Worked example: For four observations with $\mu = 10$ and $\sigma^2 = 9$, the sum has mean 40 and variance 36. Their mean has expectation 10 and variance 2.25.

Statistics · approximate confidence interval for a mean

Formula or reference: $\bar{x} \pm z\,s/\sqrt{n}$

Units and conditions: Use the critical z for the requested confidence level and check the conditions for the approximation.

Worked example: With $\bar{x} = 12$, $s = 4$ and $n = 100$, the approximate 95% interval is $12 \pm 1.96 \times 0.4$, or $(11.216, 12.784)$.

Calculus · powers and linear powers

Formula or reference: $d(x^n)/dx = nx^{(n-1)}$. $\int x^n\,dx = x^{(n+1)}/(n+1) + C$. $\int (ax+b)^n\,dx = (ax+b)^{(n+1)}/[a(n+1)] + C$.

Units and conditions: For the power antiderivatives $n \neq -1$, and $a \neq 0$ in the linear form.

Worked example: $d(x^4)/dx = 4x^3$. Also $\int x^3\,dx = x^4/4 + C$ and $\int (2x+1)^3\,dx = (2x+1)^4/8 + C$.

Calculus · exponential and logarithm

Formula or reference: $d(e^{(ax)})/dx = ae^{(ax)}$. $\int e^{(ax)}\,dx = e^{(ax)}/a + C$. $d(\ln x)/dx = 1/x$. $\int 1/x\,dx = \ln|x| + C$. $\int 1/(ax+b)\,dx = \ln|ax+b|/a + C$.

Units and conditions: Use $a \neq 0$ where dividing by a, $x > 0$ for $\ln x$, and intervals away from denominator zeros for the integrals.

Worked example: The derivative of $e^{(2x)}$ is $2e^{(2x)}$ and its antiderivative is $e^{(2x)}/2 + C$. At $x = 2$, $(\ln x)' = 1/2$. An antiderivative of $1/(3x+1)$ is $\ln|3x+1|/3 + C$.

Calculus · sine and cosine

Formula or reference: d[sin(ax)]/dx = a cos(ax), d[cos(ax)]/dx = −a sin(ax). ∫sin(ax) dx = −cos(ax)/a + C and ∫cos(ax) dx = sin(ax)/a + C.

Units and conditions: Use radians and a ≠ 0 for the antiderivative forms.

Worked example: For a = 2, the derivatives at x = 0 are 2 and 0. The antiderivatives are −cos(2x)/2 + C and sin(2x)/2 + C.

Calculus · tangent and cotangent

Formula or reference: d[tan(ax)]/dx = a sec²(ax), d[cot(ax)]/dx = −a cosec²(ax). ∫sec²(ax) dx = tan(ax)/a + C. ∫cosec²(ax) dx = −cot(ax)/a + C.

Units and conditions: Use radians and work between the corresponding poles. Use a ≠ 0 for the integral forms.

Worked example: At a = 2 and x = π/8, the tangent and cotangent derivatives are 4 and −4. The integral of cosec²(2x) is −cot(2x)/2 + C.

Calculus · secant and cosecant

Formula or reference: d[sec(ax)]/dx = a sec(ax)tan(ax). d[cosec(ax)]/dx = −a cosec(ax)cot(ax). ∫sec(ax)tan(ax) dx = sec(ax)/a + C. ∫cosec(ax)cot(ax) dx = −cosec(ax)/a + C.

Units and conditions: Use radians and stay where the functions are defined. For the integrals a ≠ 0.

Worked example: At a = 2 and x = π/8 the derivatives are 2√2 and −2√2. Differentiating sec(2x)/2 and −cosec(2x)/2 recovers the two integrands.

Calculus · inverse-trigonometric derivatives

Formula or reference: d[arcsin(ax)]/dx = a/√(1−a²x²). d[arccos(ax)]/dx = −a/√(1−a²x²). d[arctan(ax)]/dx = a/(1+a²x²).

Units and conditions: Use radians and |ax| < 1 for the first two derivatives.

Worked example: At x = 0 and a = 3 the derivatives are 3, −3 and 3.

Calculus · inverse-trigonometric integrals

Formula or reference: ∫1/√(a²−x²) dx = arcsin(x/a) + C. ∫−1/√(a²−x²) dx = arccos(x/a) + C. ∫a/(a²+x²) dx = arctan(x/a) + C.

Units and conditions: Use a > 0 and |x| < a for the first two integrands.

Worked example: For a = 2, the antiderivative of 2/(4+x²) is arctan(x/2) + C. The matching square-root forms give arcsin(x/2) + C and arccos(x/2) + C.

Calculus · product, quotient and chain

Formula or reference: (uv)′ = u′v + uv′. (u/v)′ = (vu′ − uv′)/v². dy/dx = (dy/du)(du/dx).

Units and conditions: For the quotient rule v ≠ 0.

Worked example: Differentiate x sin x to get sin x + x cos x. For x/(1+x), the quotient rule gives 1/(1+x)². For cos(x²), the chain rule gives −2x sin(x²).

Calculus · integration by parts

Formula or reference: ∫u v′ dx = uv − ∫v u′ dx

Units and conditions: Choose a differentiable u and an integrable v′.

Worked example: For ∫xeˣ dx, take u = x and v = eˣ. This gives xeˣ − eˣ + C.

Calculus · Euler’s method

Formula or reference: For y′ = f(x,y), x(next) = x + h and y(next) = y + h f(x,y), starting at the given initial point.

Units and conditions: Use the stated step h and evaluate f at the current point.

Worked example: With y′ = x + y, y(0) = 2 and h = 0.1, the first step gives x = 0.1 and y ≈ 2.2.

Calculus · parametric arc length

Formula or reference: $L = \int \sqrt{[(dx/dt)^2+(dy/dt)^2]}\, dt$ over the parameter interval.

Units and conditions: Use matching units for x and y. Length is non-negative.

Worked example: For $x = 3t$ and $y = 4t$, $0 \le t \le 2$, the speed in the parameter is 5 and arc length is 10.

Calculus · Cartesian surfaces of revolution

Formula or reference: About the x-axis: integrate $2\pi|y|\sqrt{(1+(dy/dx)^2)}\, dx$. About the y-axis: integrate $2\pi|x|\sqrt{(1+(dx/dy)^2)}\, dy$.

Units and conditions: Use the distance to the axis as the radius. Output is surface area excluding end caps.

Worked example: Rotating $y = 2$ over $0 \le x \le 3$ gives 12π square units. Rotating $x = 2$ over $0 \le y \le 3$ about the y-axis gives the same result.

Calculus · parametric surfaces of revolution

Formula or reference: Integrate $2\pi r(t)\sqrt{[(dx/dt)^2+(dy/dt)^2]}\, dt$, where $r(t) = |y(t)|$ about the x-axis or $|x(t)|$ about the y-axis.

Units and conditions: Integrate over the chosen parameter interval. Output is in square units.

Worked example: For $x = 2t$, $y = 3$, $0 \le t \le 1$, rotation about the x-axis gives surface area 12π. For $x = 3$, $y = 2t$ on the same interval, rotation about the y-axis also gives 12π.

Kinematics · acceleration

Formula or reference: $a = d^2x/dt^2 = dv/dt = v\, dv/dx = d(v^2/2)/dx$

Units and conditions: Use consistent length and time units. The position form applies when velocity can be treated as a differentiable function of position.

Worked example: For $v = 2x$ on the motion segment, $v\, dv/dx = 4x$, so at $x = 3$ the acceleration is 12 in the chosen units.

Kinematics · constant acceleration

Formula or reference: $v = u + at$. $s = ut + at^2/2$. $v^2 = u^2 + 2as$. $s = (u + v)t/2$.

Units and conditions: Choose a positive direction and use m, s and signed components.

Worked example: For $u = 2$ m/s, $a = 2$ m/s² and $t = 3$ s, the final velocity is 8 m/s and the displacement is 15 m.

Vectors · position, magnitude and derivative

Formula or reference: $r(t) = (x(t),y(t),z(t))$. $|r(t)| = \sqrt{(x^2+y^2+z^2)}$. $r'(t) = (x'(t),y'(t),z'(t))$.

Units and conditions: Differentiate each component with respect to the same parameter.

Worked example: For $r(t) = (3t,4t,t^2)$, $|r(1)| = \sqrt{26}$ and $r'(1) = (3,4,2)$.

Vectors · dot and cross products

Formula or reference: $u \cdot v = u_1v_1 + u_2v_2 + u_3v_3 = |u||v|\cos\theta$. $u \times v = (u_2v_3-u_3v_2, u_3v_1-u_1v_3, u_1v_2-u_2v_1)$.

Units and conditions: The dot product is scalar. The order fixes the direction of the cross product.

Worked example: For $u = (1,0,0)$ and $v = (1,1,0)$, $u \cdot v = 1$ and $\theta = 45°$. Their cross product is $(0,0,1)$.

Vectors · line equations

Formula or reference: Vector form $r(t) = a + tb$. Parametric coordinates are $x = a_1 + tb_1$, $y = a_2 + tb_2$ and $z = a_3 + tb_3$.

Units and conditions: Use a point a and a non-zero direction b.

Worked example: With $a = (1,2,3)$ and $b = (1,0,-1)$, $t = 2$ gives $(3,2,1)$.

Vectors · plane equations

Formula or reference: $r(s,t) = r_0 + su + tv$. Parametric coordinates follow by taking each component. Cartesian form is $ax + by + cz = d$, with normal (a,b,c).

Units and conditions: Use independent directions u and v or a non-zero normal.

Worked example: Taking $r_0 = (0,0,2)$, $u = (1,0,0)$ and $v = (0,1,0)$ gives $(x,y,z) = (s,t,2)$, so the Cartesian equation is $z = 2$.

Circular functions · Pythagorean identities

Formula or reference: $\cos^2 x + \sin^2 x = 1$. $1 + \tan^2 x = \sec^2 x$. $1 + \cot^2 x = \operatorname{cosec}^2 x$.

Units and conditions: Each identity holds where the functions on both sides are defined.

Worked example: At $x = \pi/4$, the squared sine and cosine sum to 1, while $1 + \tan^2 x$ and $1 + \cot^2 x$ both equal 2.

Circular functions · sum and difference

Formula or reference: $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$. $\tan(x \pm y) = (\tan x \pm \tan y)/(1 \mp \tan x \tan y)$.

Units and conditions: Use the upper signs together or the lower signs together. The tangent forms require non-zero denominators.

Worked example: For $x = 45°$ and $y = 30°$, $\sin(x+y) = (\sqrt{6}+\sqrt{2})/4$, $\cos(x-y) = (\sqrt{6}+\sqrt{2})/4$ and $\tan(x-y) = 2-\sqrt{3}$.

Circular functions · double angles

Formula or reference: $\sin(2x) = 2 \sin x \cos x$. $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$. $\tan(2x) = 2\tan x/(1-\tan^2 x)$.

Units and conditions: Use consistent angle units. Apply the tangent formula where both sides are defined.

Worked example: For $x = 30°$, $\sin(2x) = \sqrt{3}/2$, $\cos(2x) = 1/2$ and $\tan(2x) = \sqrt{3}$.

Circular functions · squared sine and cosine

Formula or reference: $\sin^2(ax) = [1 - \cos(2ax)]/2$. $\cos^2(ax) = [1 + \cos(2ax)]/2$.

Units and conditions: Keep angle units consistent.

Worked example: If $ax = \pi/3$, $\cos(2ax) = -1/2$. The squared sine is 3/4 and the squared cosine is 1/4.