VCE Mathematical Methods Formula Sheets

Original worked revision examples with notes on units and assumptions.

Use the board PDF below for the official sheet. Coverage of examples varies by subject.

Official reference material

Official VCAA Examination 1 formula sheet

Official VCAA Examination 2 formula sheet

What to work out beyond the sheet

Check and state an inverse function’s domain and range after solving for the inverse.

Formulas and worked examples

Mensuration · trapezium and triangle

Formula or reference: Trapezium area = (a + b)h/2. Triangle area = bc sin A/2.

Units and conditions: Use square units. h is perpendicular to the parallel sides, and A is the angle between b and c.

Worked example: A trapezium with bases 5 m and 9 m and height 3 m has area 21 m^2. A triangle with sides 6 m and 4 m enclosing 30° has area 6 m^2.

Mensuration · cylinder and cone

Formula or reference: Cylinder curved area = $2\pi rh$. Cylinder volume = $\pi r^2 h$. Cone volume = $\pi r^2 h/3$.

Units and conditions: Use square or cubic units as appropriate. Curved area excludes the end discs.

Worked example: With r = 2 cm and h = 6 cm, the cylinder’s curved area is 24π cm^2 and its volume is 24π cm^3. The cone of the same base and height has volume 8π cm^3.

Mensuration · pyramid and sphere

Formula or reference: Pyramid volume = Ah/3. Sphere volume = $4\pi r^3/3$.

Units and conditions: A is base area and h is perpendicular height. Volume uses cubic units.

Worked example: A pyramid of base area 12 cm^2 and height 6 cm has volume 24 cm^3. A sphere of radius 3 cm has volume 36π cm^3.

Calculus · powers

Formula or reference: $d(x^n)/dx = nx^{(n-1)}$. $\int x^n\,dx = x^{(n+1)}/(n + 1) + C$ for $n \neq -1$.

Units and conditions: Apply on the real domain of the expression and include C for an indefinite integral.

Worked example: $d(x^3)/dx = 3x^2$. Conversely, $\int 3x^2\,dx = x^3 + C$.

Calculus · powers of linear expressions

Formula or reference: $d[(ax + b)^n]/dx = an(ax + b)^{(n-1)}$. $\int (ax + b)^n\,dx = (ax + b)^{(n+1)}/(a(n + 1)) + C$ for $a \neq 0$ and $n \neq -1$.

Units and conditions: Work on an interval where the function is defined.

Worked example: The derivative of $(2x + 1)^3$ is $6(2x + 1)^2$. An antiderivative of $(2x + 1)^3$ is $(2x + 1)^4/8 + C$.

Calculus · exponential and logarithm

Formula or reference: d(e^(ax))/dx = ae^(ax). ∫ e^(ax) dx = e^(ax)/a + C for a ≠ 0. d(ln x)/dx = 1/x. ∫ (1/x) dx = ln x + C for x > 0.

Units and conditions: For the stated logarithm form use x > 0.

Worked example: The derivative of e^(3x) is 3e^(3x), and an antiderivative of e^(3x) is e^(3x)/3. At x = 2, the derivative of ln x is 1/2.

Calculus · trigonometric derivatives

Formula or reference: d[sin(ax)]/dx = a cos(ax). d[cos(ax)]/dx = −a sin(ax). d[tan(ax)]/dx = a/cos²(ax) = a sec²(ax).

Units and conditions: Use radians. The tangent derivative is defined where cos(ax) ≠ 0.

Worked example: For a = 2, the derivatives at x = 0 of sin(2x), cos(2x) and tan(2x) are 2, 0 and 2, respectively.

Calculus · trigonometric antiderivatives

Formula or reference: ∫ sin(ax) dx = −cos(ax)/a + C. ∫ cos(ax) dx = sin(ax)/a + C, for a ≠ 0.

Units and conditions: Use radians and include the integration constant.

Worked example: ∫ sin(3x) dx = −cos(3x)/3 + C and ∫ cos(3x) dx = sin(3x)/3 + C.

Calculus · product rule

Formula or reference: d(uv)/dx = u′v + uv′

Units and conditions: The derivative has units of output per input where applicable.

Worked example: For y = x²eˣ, y′ = 2xeˣ + x²eˣ = eˣ(2x + x²).

Calculus · quotient rule

Formula or reference: d(u/v)/dx = (vu′ − uv′)/v² for v ≠ 0.

Units and conditions: State any excluded denominator values.

Worked example: For y = x/(x + 1), y′ = [(x + 1) − x]/(x + 1)² = 1/(x + 1)² for x ≠ −1.

Calculus · chain rule

Formula or reference: dy/dx = (dy/du)(du/dx)

Units and conditions: Differentiate the outer function at u, then multiply by the derivative of u.

Worked example: For y = sin(x²), take u = x², so dy/dx = 2x cos(x²).

Calculus · Newton’s method

Formula or reference: x(next) = x(current) − f(x(current))/f′(x(current))

Units and conditions: The derivative at the current iterate must be non-zero. Convergence depends on the starting value.

Worked example: For f(x) = x² − 2, starting at x = 1 gives the next iterate 1 − (−1)/2 = 1.5.

Calculus · trapezium rule

Formula or reference: Approximate the integral by (h/2) times the sum of both endpoint heights and twice each interior height, with h = (b − a)/n.

Units and conditions: Subintervals must have equal width. The result uses input units times output units.

Worked example: For f(x) = x² from 0 to 2 with n = 2, h = 1 and the estimate is (0 + 2 × 1 + 4)/2 = 3.

Probability · complement and union

Formula or reference: $P(A) = 1 - P(A')$. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Units and conditions: Probabilities are unitless between zero and one.

Worked example: If $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, the union has probability 0.8 and A's complement has probability 0.4.

Probability · conditional probability

Formula or reference: $P(A \mid B) = P(A \cap B)/P(B)$ when $P(B) > 0$.

Units and conditions: Divide by the probability of the conditioning event.

Worked example: If $P(A \cap B) = 0.12$ and $P(B) = 0.30$, $P(A \mid B) = 0.40$.

Probability · mean and variance

Formula or reference: $\mu = E(X)$. $\mathrm{Var}(X) = \sigma^2 = E[(X - \mu)^2] = E(X^2) - \mu^2$.

Units and conditions: Mean has the units of X. Variance has squared units.

Worked example: For equally likely values 0 and 2, $\mu = 1$ and $E(X^2) = 2$, so variance is 1.

Probability · binomial coefficient

Formula or reference: $C(n,k) = n!/[k!(n - k)!]$

Units and conditions: Use integer $n \geq 0$ and k between zero and n inclusive.

Worked example: $C(5,2) = 5!/(2!3!) = 10$.

Probability · discrete distribution

Formula or reference: $P(X = x) = p(x)$. $\mu = \Sigma\, x\, p(x)$. $\sigma^2 = \Sigma\, (x - \mu)^2\, p(x)$.

Units and conditions: The probabilities sum to one.

Worked example: If X is 0 with probability 0.75 and 4 with probability 0.25, $\mu = 1$ and $\sigma^2 = 0.75 \times 1^2 + 0.25 \times 3^2 = 3$.

Probability · binomial distribution

Formula or reference: $P(X = k) = C(n,k)p^k(1 - p)\text{^}(n-k)$. $\mu = np$. $\sigma^2 = np(1 - p)$.

Units and conditions: Use n independent trials with the same success probability p.

Worked example: For four fair trials, $P(X = 2) = 6/16 = 0.375$, $\mu = 2$ and $\sigma^2 = 1$.

Probability · continuous distribution

Formula or reference: $P(a < X < b)$ is the integral of f(x) from a to b. μ is the integral of $xf(x)$, and σ^2 of $(x - \mu)^2 f(x)$, over the full support.

Units and conditions: A probability density integrates to one. Variance has squared units.

Worked example: For a uniform density of 1 on [0,1], the probability of (0, 1/2) is 1/2. Integrating x gives $\mu = 1/2$, and integrating $(x - 1/2)^2$ gives $\sigma^2 = 1/12$.

Sampling · sample proportions

Formula or reference: $\hat{P} = X/n$. $E(\hat{P}) = p$. $\mathrm{sd}(\hat{P}) = \sqrt{[p(1 - p)/n]}$.

Units and conditions: Use a binomial model for X. Proportions are unitless.

Worked example: If $n = 100$ and $p = 0.40$, the mean is 0.40 and standard deviation is about 0.049. In a sample with 43 successes the observed proportion is 0.43.

Sampling · approximate confidence interval

Formula or reference: $\hat{p} \pm z\sqrt{[\hat{p}(1 - \hat{p})/n]}$

Units and conditions: Use the critical z for the requested confidence level. Check the conditions for the normal approximation.

Worked example: With 40 successes in 100 trials and $z = 1.96$, the approximate 95% interval is 0.40 ± 0.096, or (0.304, 0.496).

More worked examples

Chain rule with an exponential

d/dx[e^(g(x))] = g′(x)e^(g(x)).

Differentiate f(x) = e^(2x−1) and find f′(½).

f′(x) = 2e^(2x−1). At x = ½ the exponent is zero, so f′(½) = 2. This is the tangent gradient at that x-value.

Check: The derivative of the inner function contributes the factor 2.

A binomial probability

For X ~ Bin(n,p), P(X=k) = C(n,k)p^k(1−p)^(n−k).

Four independent trials each have success probability 0.5. Find the probability of exactly two successes.

P(X=2) = C(4,2) × 0.5² × 0.5² = 6/16 = 0.375. Independence and constant probability justify the model.

Check: Exactly two differs from at least two; the latter also includes three and four successes.

Inverse functions and restrictions

To invert a one-to-one function, solve y = f(x) for x and swap the variables; the range becomes the inverse domain.

Find the inverse of f(x) = x² + 1 with domain x ≥ 0.

y − 1 = x². The domain restriction selects x = √(y − 1), so $f^{-1}(x) = \sqrt{x - 1}$ with domain x ≥ 1 and range y ≥ 0.

Check: Using both signs does not define a single inverse function for this restricted original domain.