HSC Maths Advanced Formula Sheet

Original worked revision examples with notes on units and assumptions.

Use the board PDF below for the official sheet. Coverage of examples varies by subject.

Official reference material

Official NESA reference PDF

Companion guide: HSC Maths Extension 1 Formula Sheet

What to work out beyond the sheet

This guide follows the reference sheet for the HSC through 2026. Check NESA's materials for later cohorts. For geometric area, split the interval at sign changes and add the positive areas. The definite integral gives signed area.

Formulas and worked examples

Measurement · arcs and sectors

Formula or reference: For degrees, $\ell = (\theta/360°) \times 2\pi r$ and $A = (\theta/360°) \times \pi r^2$. For radians, $\ell = r\theta$ and $A = r^2\theta/2$.

Units and conditions: Match the angle units to the formula. Use square units for area.

Worked example: A 60° sector with r = 3 cm has arc length π cm and area $3\pi/2$ cm². Using $\theta = \pi/3$ radians gives the same results.

Measurement · trapezium area

Formula or reference: $A = h(a + b)/2$

Units and conditions: a and b are the parallel sides, and h is perpendicular to them.

Worked example: Parallel sides of 5 m and 9 m and a height of 4 m give 28 m².

Measurement · surface areas

Formula or reference: Closed cylinder: $A = 2\pi r^2 + 2\pi rh$. Sphere: $A = 4\pi r^2$.

Units and conditions: Use square units. The cylinder formula includes both ends.

Worked example: At radius 2 cm and cylinder height 3 cm, the cylinder has area 20π cm² and the sphere has area 16π cm².

Measurement · volumes

Formula or reference: Pyramid or cone: $V = Ah/3$. Sphere: $V = 4\pi r^3/3$.

Units and conditions: Use base area A, perpendicular height h and cubic units.

Worked example: A base of 15 cm² and height 4 cm give volume 20 cm³. A sphere of radius 3 cm has volume 36π cm³.

Functions · quadratic roots

Formula or reference: For $ax^2 + bx + c = 0$, $x = [-b \pm \sqrt{(b^2 - 4ac)}]/(2a)$.

Units and conditions: $a \neq 0$. Real roots require a non-negative discriminant.

Worked example: For $x^2 - 5x + 6 = 0$, $x = (5 \pm 1)/2$, giving 2 and 3.

Relations · equation of a circle

Formula or reference: $(x - h)^2 + (y - k)^2 = r^2$

Units and conditions: Centre (h,k), radius $r > 0$. Use the same units on both axes.

Worked example: The equation $(x - 1)^2 + (y + 2)^2 = 9$ has centre (1,−2) and radius 3.

Finance · compound growth

Formula or reference: $A = P(1 + r)^n$

Units and conditions: P and A are monetary amounts. Use a decimal rate r per period.

Worked example: $1000 at 4% per year for two years becomes $1081.60.

Sequences · arithmetic term and sum

Formula or reference: $T(n) = a + (n - 1)d$. $S(n) = n[2a + (n - 1)d]/2 = n(a + \ell)/2$, where ℓ is the last term.

Units and conditions: n is a positive integer. a, d, ℓ and the sum use the same units.

Worked example: For 4, 7, 10, 13, 16 the fifth term is 16 and the sum is $5(4 + 16)/2 = 50$.

Sequences · geometric term and sums

Formula or reference: T(n) = ar^(n−1). $S(n) = a(1 - r^n)/(1 - r) = a(r^n - 1)/(r - 1)$ for $r \neq 1$. $S_\infty = a/(1 - r)$ for $|r| < 1$.

Units and conditions: The common ratio r is unitless. An infinite sum requires convergence.

Worked example: For $a = 8$ and $r = 1/2$, $T(3) = 2$, $S(3) = 14$ and $S_\infty = 16$.

Logarithms · inverse and change of base

Formula or reference: $\log_a(a^x) = x$ and a^(log$_a$ x) = x. $\log_a x = \log_\beta x/\log_\beta a$. $a^x = e^{(x \ln a)}$.

Units and conditions: Bases are positive and different from 1. Logarithm arguments are positive.

Worked example: $\log_2 8 = 3$, which can also be evaluated as $\ln(8)/\ln(2)$. Then $2^3 = e^{(3 \ln 2)} = 8$.

Trigonometry · right triangles and reciprocals

Formula or reference: sin A = opp/hyp, cos A = adj/hyp, tan A = opp/adj. sec A = 1/cos A, cosec A = 1/sin A, cot A = cos A/sin A.

Units and conditions: Ratios are unitless. Denominators must be non-zero.

Worked example: For an angle opposite the side of length 3 in a 3–4–5 triangle, sin A = 3/5, cos A = 4/5, tan A = 3/4, sec A = 5/4, cosec A = 5/3 and cot A = 4/3.

Trigonometry · standard triangles and identity

Formula or reference: 45°–45°–90° sides are in ratio 1:1:√2. 30°–60°–90° sides are 1:√3:2. $\cos^2 x + \sin^2 x = 1$.

Units and conditions: Match each side to the opposite angle.

Worked example: sin 30° = 1/2 and cos 30° = √3/2. Their squares add to 1. At 45°, both ratios equal 1/√2.

Trigonometry · non-right triangles

Formula or reference: $A = ab \sin C/2$. $a/\sin A = b/\sin B = c/\sin C$. $c^2 = a^2 + b^2 - 2ab \cos C$ and $\cos C = (a^2 + b^2 - c^2)/(2ab)$.

Units and conditions: Match each side to its opposite angle. The area uses the angle between a and b.

Worked example: For sides a = 3 cm, b = 4 cm with C = 90°, the area is 6 cm² and c = 5 cm. The sine rule gives sin A = 3/5.

Statistics · standard score and outlier fences

Formula or reference: $z = (x - \mu)/\sigma$. Lower and upper fences are $Q_1 - 1.5$ IQR and $Q_3 + 1.5$ IQR.

Units and conditions: z is unitless. The fences have the data’s units.

Worked example: At $x = 74$, $\mu = 62$ and $\sigma = 8$, $z = 1.5$. If $Q_1 = 58$ and $Q_3 = 66$, the fences are 46 and 78.

Statistics · normal distribution bands

Formula or reference: About 68%, 95% and 99.7% of a normal population lie within 1, 2 and 3 standard deviations of the mean, respectively.

Units and conditions: Percentages are approximations for a normal distribution.

Worked example: For a normal distribution with mean 50 and standard deviation 4, the two-standard-deviation interval is 42 to 58 and contains about 95%.

Statistics · mean and variance

Formula or reference: $E(X) = \mu$. $\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$.

Units and conditions: The mean uses the original units and variance uses squared units.

Worked example: If X is 0 or 4 with equal probability, $E(X) = 2$ and $E(X^2) = 8$, so $\operatorname{Var}(X) = 4$.

Probability · independent, combined and conditional events

Formula or reference: For independent A and B, $P(A \cap B) = P(A)P(B)$. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. $P(A \mid B) = P(A \cap B)/P(B)$ for $P(B) > 0$.

Units and conditions: Probabilities are unitless. Only the product expression requires independence.

Worked example: If independent events have probabilities 0.5 and 0.4, their intersection is 0.2, union is 0.7, and $P(A \mid B)$ is 0.5.

Probability · continuous random variables

Formula or reference: If a is the lower endpoint of the support, $P(X \leq r) = \int$ from a to r of $f(x)\,dx$. $P(a < X < b) = \int$ from a to b of $f(x)\,dx$.

Units and conditions: Use a valid density f and integrate over the event interval.

Worked example: A uniform density of 1/4 from 0 to 4 gives $P(X \leq 1) = 1/4$ and $P(1 < X < 3) = 1/2$.

Calculus · derivative rules

Formula or reference: $d[f(x)^n]/dx = nf'(x)f(x)^{(n-1)}$. $(uv)' = u'v + uv'$. $(u/v)' = (vu' - uv')/v^2$. $dy/dx = (dy/du)(du/dx)$.

Units and conditions: Use each rule where its component functions are differentiable and $v \neq 0$ in the quotient rule.

Worked example: Differentiating $(3x + 1)^2$ gives $6(3x + 1)$. For $x^2(x + 1)$, the product rule gives $3x^2 + 2x$. For $x/(x + 1)$, the quotient rule gives $1/(x + 1)^2$.

Calculus · trigonometric derivatives

Formula or reference: $d[\sin f]/dx = f' \cos f$. $d[\cos f]/dx = -f' \sin f$. $d[\tan f]/dx = f' \sec^2 f$.

Units and conditions: Use radians and avoid tangent poles.

Worked example: At $x = 0$, the derivatives of $\sin(2x)$, $\cos(2x)$ and $\tan(2x)$ are 2, 0 and 2.

Calculus · exponential and logarithmic derivatives

Formula or reference: $d(e^f)/dx = f'e^f$. $d(a^f)/dx = (\ln a)f'a^f$. $d(\ln f)/dx = f'/f$. $d(\log_a f)/dx = f'/(f \ln a)$.

Units and conditions: Logarithm arguments must be positive. Use a valid positive base different from 1.

Worked example: At $x = 1$, the derivatives of $e^{(2x)}$, $2^{(2x)}$, $\ln(2x)$ and $\log_2(2x)$ are $2e^2$, $8 \ln 2$, 1 and $1/\ln 2$.

Calculus · power antiderivatives

Formula or reference: $\int f'(x)f(x)^n\,dx = f(x)^{(n+1)}/(n + 1) + C$ for $n \neq -1$.

Units and conditions: Use an interval where the functions are defined. Include the integration constant.

Worked example: $\int 3x^2\,dx = x^3 + C$. With $f(x) = 2x + 1$, $\int 2(2x + 1)^3\,dx = (2x + 1)^4/4 + C$.

Calculus · trigonometric antiderivatives

Formula or reference: ∫ f′ sin f dx = −cos f + C. ∫ f′ cos f dx = sin f + C. ∫ f′ sec² f dx = tan f + C.

Units and conditions: Use radians and work between poles for the last expression.

Worked example: With f(x) = 3x, antiderivatives of 3 sin(3x), 3 cos(3x) and 3 sec²(3x) are −cos(3x), sin(3x) and tan(3x), respectively, each plus C.

Calculus · exponential and logarithmic antiderivatives

Formula or reference: ∫ f′e^f dx = e^f + C. ∫ f′a^f dx = a^f/ln a + C. ∫ f′/f dx = ln|f| + C.

Units and conditions: Use a > 0 with a ≠ 1. Work where f is non-zero in the logarithmic form.

Worked example: With f(x) = 2x, antiderivatives of 2e^(2x), 2 × 3^(2x) and 1/x are e^(2x), 3^(2x)/ln 3 and ln|x|, each plus C.

Calculus · trapezoidal approximation

Formula or reference: For n equal strips, multiply (b − a)/(2n) by the sum of the endpoint heights and twice each interior height.

Units and conditions: Use equal subintervals. The integral has units of input times output.

Worked example: On [0,2] for f(x) = x², two strips give (0 + 2 × 1 + 4)/2 = 3.

More worked examples

Chain rule

For y = [g(x)]^n, dy/dx = n[g(x)]^(n−1)g′(x).

Differentiate y = (3x + 1)^4.

The outside derivative is 4(3x + 1)^3 and the inside derivative is 3. Multiplying gives y′ = 12(3x + 1)^3.

Check: Omitting the inside derivative gives a result too small by a factor of three.

Signed area versus total area

A definite integral adds signed contributions; split at zeros to find total area.

Find the total area between y = x and the x-axis for −1 ≤ x ≤ 1.

The integral from −1 to 1 of x is zero, but the area is not. Each triangle has area ½ × 1 × 1, so the total area is 1 square unit. Equivalently integrate −x on [−1,0] and x on [0,1].

Check: A zero definite integral can reflect cancellation of equal positive and negative contributions.

Exponential rate of change

If P(t) = Ae^(kt), then P′(t) = kAe^(kt).

A model has P(t) = 100e^(0.2t), with t in hours. Find its initial rate of change.

P′(t) = 20e^(0.2t), so P′(0) = 20 units per hour. P(0) = 100 is the starting amount, not the rate.

Check: Differentiate before substituting if the question asks for a rate; include the time unit.